Regularisation by Noise

Vienna, 14–17th April 2025

Organising Committee:

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Monday, 14.4.2025

On strong solution for one multidimensional SDE

Alexander Veretennikov

TBA

On uniqueness and regularity results of stochastic differential equations on the plane with rough drift

Olivier Pamen

In this talk, we discuss the existence, uniqueness, and regularization by noise for stochastic differential equations (SDEs) on the plane. These equations can also be interpreted as quasi-linear hyperbolic stochastic partial differential equations (HSPDEs). More specifically, we address path-by-path uniqueness for multidimensional SDEs on the plane, under the assumption that the drift coefficient satisfies a spatial linear growth condition and is componentwise non-decreasing. In the case where the drift is only measurable and uniformly bounded, we show that the corresponding additive HSPDE on the plane admits a unique strong solution that is Malliavin differentiable. Our approach combines tools from Malliavin calculus with variational techniques originally introduced by Davie (2007), which we non-trivially extend to the setting of SDEs on the plane. This talk is based on a joint works with A. M. Bogso, M. Dieye and F. Proske.

Energy solutions to (super-)critical S(P)DEs

Lukas Gräfner

We study SDEs driven by Brownian motion and distributional drift b on $\mathbb{T}^d$ or $\mathbb{R}^d$ with $d \geq 2$. We work in a scaling-supercritical regime using energy solutions and recent ideas for generators of singular SPDEs. We mainly focus on divergence-free b, but allow for scaling-critical, non-divergence perturbations. Roughly speaking we prove weak well-posedness of energy solutions X for any Lebesgue-absolutely continuous initial law and $b \in L^p_t B^{-\gamma}_{p,1}$ with $p > \frac{1}{1-\gamma}$, $\gamma \in [0,1]$, $p \in (2, \infty]$. We can extend this by allowing a blow-up of $\|b\|_{L^p_t B^{-\gamma}_{p,1}}$ around some small space-time singularity set, but for time-dependent b we have to assume X to be of a certain Hölder regularity to make the equation well-posed. In this way we can find for any $p > 2$ a dimension d and $b \notin B^{-1}_{p,2}$ such that weak well-posedness holds for (Hölder-regular) energy solutions and drift b. This talk is based on joint work with Nicolas Perkowski.

Regularization by noise and approximations of singular kinetic SDEs

Chengcheng Ling

We study the strong convergence of a generic tamed Euler-Maruyama (EM) scheme for the kinetic type stochastic differential equation (SDE) (also known as second order SDE) driven by α-stable type noise with $\alpha \in (1, 2]$. We show that when the drift exhibits a relatively low regularity: anisotropic β-Hölder continuity with $\beta > 1 - \frac{\alpha}{2}$, the corresponding tamed EM converges with a convergence rate $(\frac{1}{2} + \frac{\beta}{\alpha(1+\alpha)} \wedge \frac{1}{2})$, which aligns with the results of first-order SDEs [DBG22].

This talk is based on the work arXiv:2409.05706 (joint with Zimo Hao and Khoa Lê) and the work arXiv:2412.05142.

p-Brownian motion and the p-Laplacian

Michael Röckner

In this talk we shall present the construction of a stochastic process, which is related to the parabolic p-Laplace equation in the same way as Brownian motion is to the classical heat equation given by the (2-) Laplacian.

Joint work with:
1) Viorel Barbu, Al.I. Cuza University and Octav Mayer Institute of Mathematics of Romanian Academy, Iaşi, Romania
2) Marco Rehmeier, Faculty of Mathematics, Bielefeld University, Germany

References:
[1] V. Barbu, M. Rehmeier, M. Röckner: arXiv:2409.18744
[2] V. Barbu, M. Röckner: Springer LN in Math. 2024
[3] M. Rehmeier, M. Röckner: arXiv:2212.12424

Kinetic McKean SDEs with singular coefficients

Elena Issoglio

In this talk we consider a McKean-Vlasov type SDE with drift in anisotropic Besov spaces with negative regularity and with degenerate diffusion matrix under the weak Hörmander condition. The classical example of degeneracy is the kinetic equation, that describes the evolution of a particle in the phase-space, where the position dynamics is the integral of the velocity, while the velocity dynamics is governed by a stochastic differential equation. These equations are therefore more tricky than usual from a regularization by noise perspective, since the noise is not present in all dimensions. The key is to rescale differently the various directions and use an anisotropic distance underlying everything.

We give a meaning to the McKean-Vlasov degenerate singular equation via a suitable martingale problem, thus a key tool is the study of the corresponding singular anisotropic Kolmogorov equation. A solution can be found via a 'linearization' procedure to remove the dependence on the law of the unknown, and for this we use the solution to the singular anisotropic nonlinear Fokker-Planck equation.

This talk is based on a joint work with S. Pagliarani (Bologna), D. Trevisani (A Coruna) F. Russo (ENSTA).

Towards the Krylov-Röckner condition for SDEs driven by fractional Brownian motion

Oleg Butkovsky

Based on joint works with S. Gallay, K. Le, and T. Matsuda. In a seminal work, Krylov and Röckner extended the results of Zvonkin and Veretennikov and showed that the stochastic differential equation

$$dX_t = b(t, X_t)dt + dW_t$$

has a unique strong solution if $b \in L_q([0, T], L_p(\mathbb{R}^d))$ and

$$\frac{2}{q} + \frac{d}{p} < 1.$$

More recently, Krylov showed that weak existence holds under the weaker condition

$$\frac{1}{q} + \frac{d}{p} < 1.$$

How do these conditions extend to the case where the driving noise is not Brownian but fractional Brownian motion? The main method used in the Brownian case—the Zvonkin transformation—is no longer available in the fractional setting. We present recent progress in this direction, including counterexamples that demonstrate the sharpness of some of the obtained results.

Numerical approximation of stochastic (partial) differential equations with distributional drift

Alexandre Richard

In this talk, we deal with the numerical approximation for two classes of equations. First we consider the d-dimensional SDE $dX_t = b(X_t) + dB_t$, where b is singular (typically a distribution) and B is a fractional Brownian motion with Hurst parameter smaller than 1/2. Introducing an Euler scheme with mollified drift and using the regularisation effect of the noise, we quantify the strong error of approximation between this scheme and X. An optimization in the mollification permits to obtain a rate of convergence that recovers the known rate in the case of bounded drift, and extends it to drifts with negative regularity. Second, a similar study will be presented for the stochastic heat equation with distributional term, considering a tamed Euler scheme in time, with finite differences in space. Joint work with E.M. Haress and L. Goudenège.

Tuesday, 15.4.2025

Global well-posedness by transport noise of 3D Navier-Stokes equations with small hyperviscosity

Antonio Agresti

Global well-posedness of 3D Navier-Stokes equations (NSEs) is one of the biggest open problems in modern mathematics. A long-standing conjecture in stochastic fluid dynamics suggests that physically motivated noise can prevent (potential) blow-up of solutions of the 3D NSEs. In this talk, I will review recent developments on the topic and discuss the solution to this problem in the case of the 3D NSEs with small hyperviscosity, for which the global well-posedness in the deterministic setting remains as open as for the 3D NSEs. The key new ideas are related to the use L^p-techniques in the context of scaling limits for SPDEs, where L^p refers to the time integrability. The case without hyperviscosity can be seen as critical for our methods, and the extension of our techniques to this case poses new challenges at the intersection of harmonic and stochastic analysis, which, if time permits, will be discussed at the end of the talk.

Quantitative Propagation of Chaos for Singular Interacting Particle Systems Driven by Fractional Brownian Motion

Avi Mayorcas

Systems of interacting particles are ubiquitous in physics, biology, chemistry, computer science and the social sciences. In many applications, the desired interaction potential is a highly singular function or distribution, making well-posedness, mean field convergence, and other quantitative results challenging to obtain. In this talk I will present recent results, obtained with L. Galeati and K. Lê, on interacting particle systems driven by i.i.d. fractional Brownian motions, subject to irregular, possibly distributional, pairwise interactions. We show propagation of chaos and mean field convergence to the law of the associated McKean–Vlasov equation, as the number of particles $N \to \infty$, with quantitative sharp rates of order $\sqrt{N}$. Our results hold for a wide class of possibly time-dependent interactions, which are only assumed to satisfy a Besov-type regularity, related to the Hurst parameter $H \in (0, +\infty) \setminus \mathbb{N}$ of the driving noises. In particular, as H decreases to 0, interaction kernels of arbitrary singularity can be considered, a phenomenon frequently observed in regularization by noise results. Our proofs rely on a combination of Sznitman's direct comparison argument with stochastic sewing techniques.

On conditional uniqueness for stochastic Navier-Stokes equations

István Gyöngy

Stochastic Navier-Stokes equations are considered in the whole space for dimension greater than two. The famous Ladyzhenskaya-Prodi-Serrin condition on conditional uniqueness of Leray-Hopf week solutions to deterministic Navier-Stokes equations is generalised and extended to stochastic Navier-Stokes equations, and a theorem on conditional regularity is also presented. The talk is based on joint work with Nicolai Krylov.

Regularization by noise for some modulated dispersive PDEs

Tristan Robert

In this talk, we will consider nonlinear dispersive PDEs where a deterministic noise is added as a distributional time coefficient in front of the dispersion. Despite the roughness of the noise term, we will see that any semilinear dispersive PDE with this noise term is well-posed at least in the same range of regularity as its noiseless counterpart, as soon as well-posedness relies on linear space-time estimates. Building on previous works on this model, we will also observe several regularization by noise phenomena provided that the noise is irregular enough: large data global well-posedness for focusing mass-critical equations, well-posedness at super-critical regularity for strongly non-resonant equations through improved multilinear estimates, and improvement on the Cauchy theory for Kadomtsev-Petviashvili equations through short-time multilinear estimates on longer time scales.

Wednesday, 16.4.2025

$\sqrt{\log t}$ - superdiffusivity for a Brownian particle in the curl of the 2d GFF

Fabio Toninelli

The present work is devoted to the study of the large time behaviour of a critical Brownian diffusion in two dimensions, whose drift is divergence-free, ergodic and given by the curl of the 2-dimensional Gaussian Free Field. We prove the conjecture, made in [B. Tóth, B. Valkó, J. Stat. Phys., 2012], according to which the diffusion coefficient $D(t)$ diverges as $\sqrt{\log t}$ for $t \to \infty$. Starting from the fundamental work by Alder and Wainwright [B. Alder, T. Wainwright, Phys. Rev. Lett. 1967], logarithmically superdiffusive behaviour has been predicted to occur for a wide variety of out-of-equilibrium systems in the critical spatial dimension $d = 2$. Examples include the diffusion of a tracer particle in a fluid, self-repelling polymers and random walks, Brownian particles in divergence-free random environments, and, more recently, the 2-dimensional critical Anisotropic KPZ equation. Even if in all of these cases it is expected that $D(t) \sim \sqrt{\log t}$, to the best of the authors' knowledge, this is the first instance in which such precise asymptotics is rigorously established.

On weak uniqueness of the Itô equations with singular drift

Nicolai Krylov

We consider the Itô equation $dx_t = b(x_t)\,dt + \sigma(x_t)\,dw_t$ with b from a Morrey class, which contains b such that $|b| \leq |x|^{-1}$. The main question is how bad uniformly nondegenerate, bounded σ could be to still "stabilize" the equation in the sense to make it solvable. We present an answer in terms of weak solvability and weak uniqueness.

Regularity of the conditional densities for singular fractional SDEs

Lucio Galeati

We consider multidimensional SDEs with singular drift, driven by additive fractional Brownian motion (fBm). Under appropriate regularity assumptions, such equations are known to be solvable in a strong sense, thanks to modern tools like the Stochastic Sewing Lemma (SSL). However, due to the singularity of the drift and the non-Markovian nature of the noise, many standard tools to estimate the density of the law of the solution are not available anymore; conditional estimates are even harder to attain. In this talk I will present several results in this direction, based on a combination of duality arguments, sewing techniques, Romito's lemma and Girsanov transform. As a consequence, we provide novel existence and uniqueness results for McKean-Vlasov equations driven by fBm with convolutional drift, thanks to a regularity bootstrapping procedure. Based on an ongoing joint work with Lukas Anzeletti, Alexandre Richard and Etienne Tanré.

Regularization by noise for rough differential equations driven by Gaussian rough paths

Rémi Catellier

In this talk, I will present the main results of the paper "Regularization by noise for rough differential equations driven by Gaussian rough paths" (joint work with Romain Duboscq). The paper concerns the regularization by noise for rough differential equations (RDEs) with drift, driven by Gaussian geometric rough paths. The central result shows that, under natural conditions — notably local non-determinism of the noise and uniform ellipticity of the diffusion coefficient — one can obtain path-by-path uniqueness of solutions, even when the drift term has very low regularity.

The approach combines a flow transformation inspired by Davie's work and advanced tools from Malliavin calculus adapted to the Gaussian rough path setting and allows to treat a wide class of Gaussian processes.

Anomalous Regularization in Kazantsev-Kraichnan Model

Marco Bagnara

We investigate a passive vector field which is transported and stretched by a divergence-free Gaussian velocity field, delta-correlated in time and poorly correlated in space (spatially nonsmooth). Although the advection of a scalar field (Kraichnan's passive scalar model) is known to enjoy regularizing properties, the potentially competing stretching term in vector advection may induce singularity formation. We establish that the regularization effect is actually retained in certain regimes. While this is true in any dimension $d \geq 3$, it notably implies a regularization result for linearized 3D Euler equations with stochastic modeling of turbulent velocities, and for the induction equation in magnetohydrodynamic turbulence. The presentation is based on a joint work with Francesco Grotto and Mario Maurelli.

Global well-posedness of a stochastic nonlinear heat equation with constraints of finite codimension

Ashish Bawalia

We examine a stochastic nonlinear heat equation in any dimension $d \geq 1$ driven by a Gaussian noise in the Stratonovich form along with a constraint on the L^2-norm of the solution. The existence of an $H_0^1 \cap L^p-$valued ($2 \leq p < \infty$) martingale solution is shown. Moreover, we have shown that this solution is invariant in a Hilbertian manifold , in particular unit sphere, that is, if the initial data is in , then all its corresponding trajectories stay in . Finally, the pathwise uniqueness of the solution is proved, which concludes the existence of a strong solution via a Yamada-Watanabe-type result.

Long-time behaviour of singular SPDEs

El Mehdi Haress

We study existence and uniqueness for a one-dimensional quasi-linear stochastic heat equation driven by a space-time white noise and a distributional drift. Assuming that the regularity of the distributional drift is strictly greater than -1, we show that the SPDE has a unique weak solution in a class of Hölder-continuous functions in time. We also prove uniform-in-time bounds on the moments of the solution. The proof rely on stochastic sewing techniques, especially to deduce new regularisation properties of the Ornstein-Uhlenbeck process.

An extended variational setting for critical SPDEs with Levy noise

Fabian Germ

The critical variational setting was recently introduced and shown to be applicable to many important SPDEs not covered by the classical variational setting. We introduce a flexibility in the range space for the nonlinear drift term, due to which certain borderline cases can now also be included. In addition to this, we allow the drift to be singular in time. In this talk, I will give an overview of our results with a focus on the applications for which our results yield an improvement. An example of this is the Allen-Cahn equation in dimension two in the weak setting.

Regularisation by multiplicative noise for reaction-diffusion equations

Teodor Holland

We consider the stochastic reaction-diffusion equation in 1+1 dimensions driven by multiplicative space-time white noise, with a distributional drift belonging to a Besov-Hölder space with any regularity index larger than -1. We assume that the diffusion coefficient is a regular function which is bounded away from zero. By using a combination of stochastic sewing techniques and Malliavin calculus, we show that the equation admits a unique solution.

Quantitative relative entropy estimates for interacting particle systems with common noise

Paul Nikolaev

We derive quantitative estimates proving the conditional propagation of chaos for large stochastic systems of interacting particles subject to both idiosyncratic and common noise. We obtain explicit bounds on the relative entropy between the conditional Liouville equation and the stochastic Fokker–Planck equation with an bounded and square integrable interaction kernel, extending far beyond the Lipschitz case. Our method relies on reducing the problem to the idiosyncratic setting, which allows us to utilize the exponential law of large numbers. As a byproduct, we demonstrate that when conditioned on the common noise, the relative entropy remains unaffected by it.

A Stochastic Partial Differential Equation with Discontinuous Drift

Johanna Weinberger

In this talk, we study continuous random field solutions to the one-dimensional SPDE

$$\partial_t u_t = \frac{1}{2}\Delta u_t + h(u_t) + \sqrt{u_t}\dot{W},$$

where $\dot{W}$ is space-time white noise and h is a function satisfying certain structural conditions. These assumptions allow for drift terms that are merely Hölder continuous with exponent $\alpha \in (0,1)$, as well as certain discontinuous functions.

Our main result establishes weak existence and uniqueness of solutions using the duality method. We extend the classical duality relation of super-Brownian motion by introducing solutions to the log-Laplace equation with multiplicative space-time Lévy noise as the dual process and extending the duality function. The construction of the dual process relies on the theory of initial traces for the parabolic equation

$$\partial_t v_t = \frac{1}{2}\Delta v_t - \frac{1}{2}v_t^2.$$

A key application of our framework is the case

$$h(x) = \mathbb{1}_{x=0},$$

which highlights differences in the regularizing effects of multiplicative space-time white noise versus multiplicative Brownian motion. Notably, while the SDE

$$x_t = \mathbb{1}_{x_t=0}dt + \sqrt{x_t}dB_t, \quad x_0 = 0$$

where B is a standard Brownian motion, has no solution, we prove weak well-posedness for the corresponding one-dimensional SPDE. The talk is based on joint work with Leonid Mytnik.

Thursday, 17.4.2025

Stochastic transport equation with Lévy noise

Enrico Priola

We study the linear transport equation with a globally Hölder continuous and bounded vector field driven by a non-degenerate Lévy noise of α-stable type:

$$\frac{\partial u(t,x)}{\partial t} + (b(x)\cdot\nabla)u(t,x)\,dt + \sum_{i=1}^{d} e_i \cdot \nabla u(t-,x) \diamond dL_t^i = 0,\ t>0;$$

$$u(0,x) = u_0(x), \quad x \in R^d,$$

where the initial data u_0 is a bounded Borel function and the stochastic integration is understood in the Marcus form. Assuming also an integrability condition on the divergence of b we mainly consider $\alpha \in [1,2)$. In particular we prove well-posedness theorems for L^∞-weak solutions. We can even prove a uniqueness result for L^∞-weak solutions in the case of $\alpha \in (0,1)$ assuming in addition that $L_t = (L_t^1, \cdots, L_t^d),\ t \geq 0$ is rotationally invariant. This shows regularization by noise phenomena with Lévy noises. Indeed uniqueness is restored by the presence of L since without the noise term the previous equation has in general many solutions. This is a joint work with Zdzislaw Brzezniak, Zhu Jiahui and Jianliang Zhai.

Existence and uniqueness by Kraichnan noise for 2D Euler equations with unbounded vorticity

Michele Coghi

We consider the 2D Euler equations on $\mathbb{R}^2$ in vorticity form, with unbounded initial vorticity, perturbed by a suitable non-smooth Kraichnan transport noise, with regularity index $\alpha \in (0,1)$. We show weak existence for every $\dot{H}^{-1}$ initial vorticity. Thanks to the noise, the solutions that we construct are limits in law of a regularized stochastic Euler equation and enjoy an additional $L^2([0,T], H^{-\alpha})$ regularity. For every $p > 3/2$ and for certain regularity indices $\alpha \in (0, \frac{1}{2})$ of the Kraichnan noise, we also show pathwise uniqueness for every L^p initial vorticity. This result is not known without noise.

Regularisation by Gaussian rough path lifts of fractional Brownian motions

Konstantinos Dareiotis

In this talk we will discuss the solvability of rough differential equations with distributional drift driven by the Gaussian rough path lift of fractional Brownian motion with Hurst parameter $H \in (1/3, 1/2)$. Under the assumption that the noise coefficient is uniformly elliptic and sufficiently regular and that the drift is a distribution in the Hölder-Besov space $\mathcal{C}^\alpha$ with $\alpha > 1 - 1/(2H)$, we will see that the equation admits a unique strong solution. The condition $\alpha > 1 - 1/(2H)$ matches the one of the additive noise setting from [Catellier-Gubinelli, 2016] thereby providing a complete multiplicative analogue. This is joint work with M. Gerencsér, K. Lê, and C. Ling.

Weak approximation error for some singular dynamics

Nikolas Perkowski

I will present two quantitative approximation results for singular dynamics, that are spiritually related by both relying on new regularity estimates for singular infinite-dimensional Kolmogorov equations. More precisely, I will discuss the approximation of interacting particle systems by Dean-Kawasaki type SPDEs, and the approximation of the KPZ/Burgers equation by its Galerkin approximation. Based on joint works with Ana Djurdjevac, Lukas Gräfner, Helena Kremp and Xiaohao Ji.